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MAT 204, Professor Yarmola

Dean's Date Assignment

Least Squares Analysis of the Attributes of Characters in Super Smash Bros Ultimate, as Compared to a Tier List of these Characters

Project description:

We examined the video game Super Smash Bros Ultimate, specifically the relationship between character attributes and a tier list of all characters based on voting by professional players. We found how characters' moves and other statistics correlate with their tier list ranking using a least squares model.

We used least squares in order to calculate correlation coefficients between two data sets that evaluated all 73 characters differently. One data set analyzed the objective attributes of each character based on in-game data. Another data set is a subjective ranking of all characters based on the opinions of professional players. Our data analysis therefore determines the accuracy of this tier list, and what attributes are common among characters generally seen as better.

Specifically, our results will show whether better characters tend to have better offensive, defensive, and movement statistics. Each of these categories includes many attributes, each of which will have its own correlation coefficient. This will tell us what moves should be used by players attempting to win with the best characters, since professional players tend to rank characters that place higher in tournaments as better. Also, it will tell us which statistic is most important in determination of a good character: offense, defense, or movement.

<u>Overview of algorithm:</u>

At first, we tried using SVD to find an objective tier list based on the stats of each character. Our plan was to find the eigenvalues for each character and see which ones were the largest. When we compiled our data and applied SVD, we realized that the eigenvalues in the diagonal array were already ordered in a decreasing order, so unless we manually programmed an SVD algorithm, we could not find which eigenvalue corresponded to which character.

Then we looked at the least squares method and found that we already had the relevant matrices and vectors. We had to apply least squares three times for our three categories (offense, defense, and movement). The resulting vector for the three least squares models was the characters' tier list rankings while the coefficient matrices were the corresponding data for each category. Our coefficient matrices are described in further detail in our data section.

We then used the least squares formula of (A^t)(A)(x) = (A^t)(b) where A was the coefficient matrix and b was the resulting vector. A was a 73 x n matrix where the rows represented the 73 characters and the columns represented the n number of attributes we examined. For the offense matrix, n = 13. For defense, n = 10. For movement, n = 7. The resulting (A^t)(A) was a n x n matrix. We then solved for x by computing (A^t)(A)\(A^t) in MATLAB, which made use of the inverse of (A^t)(A) or the pseudoinverse of A, depending on whether A was full rank.

Discussion of linear algebra used:

A least squares model is used in regression analysis to approximate solutions to an overdetermined system, which is a system where there are more equations than unknowns. It allows you to input data points corresponding to variables and a result vector, and computes weights that can be multiplied to the variables to obtain the result vector. In essence, the least squares solutions is equal to ((A^t) (A)) ^ (-1) (A^t) (b) where A is the coefficient matrix and b is the resulting vector.

Least squares was originally applied to astronomy and geodesy as scientists tried to find ways to effectively navigate the oceans. This method evolved from a combinations of different averaging and probability techniques. Gauss stated that the least squares method is optimal in a linear model where the errors have a mean of zero, which is now known as the Gauss-Markov theorem.

The most important application of least squares is in data fitting, which it finds a line of best fit by minimizing the sum of the squared residuals, which are the differences between the observed value and the data point value. In addition to being widely used in statistics, least squares is also used in principal component analysis, which takes possibly correlated variables and separates them into linearly independent variables.

Data:

We split our data analysis into three categories: offense, defense, and movement. Offense included the damage per second (or DPS) of each of thirteen standard attacks unique to each of the 73 characters. We calculated these values from an online database, which presented the damage (in percent, an in-game unit) and frame data (there are sixty frames in one second) of each attribute for each character. Therefore, DPS=(damage/frames)*60. Since all of our values were calculated in DPS and greater DPS is beneficial to a character, we did not have to scale our values.

Defense included the frame data of nine defensive options (spot dodges, rolls, and air dodges all measured in frames) unique to each character, and a weight statistic (an in-game unit) for each character. Since one of our values was calculated in a different unit (weight), and the other values are harmful to a character (more frames makes a character slower), we had to scale our values. We multiplied the nine frame data rows by -1 in order to make greater values beneficial to a character. We decided to leave the weight statistic as is because its mean value was similar to the mean values of the other defensive statistics.

Finally, movement included the frame data of three grab attacks unique to each character, and four other movement statistics (run speed, air speed, vertical recovery, and horizontal recovery, all of which are in-game units). (Recovery refers to the maximum distance a character can travel before all of its moves are exhausted in a given direction, for example vertical recovery is the greatest height a given character can reach by itself.) Since some of our values were calculated in units other than frames (other movement statistics), and some of our values are harmful to a character (more frames makes a character slower), we had to scale our values. We multiplied the three frame data rows by -1 in order to make greater values beneficial to a character. We scaled the other rows by a fraction (mean grab value divided by mean value of the specified row) in order to make the correlation coefficients proportional.

Our data is attached in three separate Google Sheets documents. Note that the values highlighted in yellow are our x vectors.

Conclusion:

By applying least squares to our three models, we found three resulting coefficient correlation vectors. These vectors represent the correlation between the moves that the coefficients correspond to and the resulting tier list rating. We found that most moves have a small correlation (< |0.1|) to the tier list rating while a few moves had significant positive correlations (> 0.4), and there were even some moves that had a significant negative correlation (< -0.7).

Based on these results, we concluded that offensive statistics have the greatest correlation to the tier list, with an average correlation coefficient of ~0.0436. The best characters are able to hit hard and fast, implying that an offensive play style is rewarding with better characters. The best moves to use when playing a good character are forward smash (0.459), down tilt (0.0647), and up tilt (0.0495).

Defensive statistics have a negative correlation to the tier list, with an average correlation coefficient of -0.0272. This implies that characters with good defensive options are unable to compensate in terms of offense and movement, so are generally worse. Better characters should make use of forward roll (0.4570), but not backward roll (-0.7320).

Movement statistics have a smaller positive correlation to the tier list, with an average correlation coefficient of 0.007175. Characters that are more mobile are generally better, but this is much less significant than offense. The best movement options for better characters are grab

(0.3465) and air speed (0.02). Since run speed is negatively correlated with the tier list (-0.0007), better characters should stick to the air.

Overall, this tier list seems to be accurate, but there is room for improvement. Professional players seem fixated on offensive potential, often ignoring defensive potential. In further analysis, we would compare the objective in-game data to tournament results, in order to test which attributes are beneficial in practice.

Our Google Sheets with data and calculations:

https://docs.google.com/spreadsheets/d/1IBfWRz3gMeoEANicAMj6A46RdJ60gQDxZcDUHPRo9SY/edit#gid=0

https://docs.google.com/spreadsheets/d/19SOHWrWbWKm3SpQg_qwy7Bo-5n-JIAX46pCkaw4440U/edit#gid=0

https://docs.google.com/spreadsheets/d/1fd0VR-vE6chkquMKFuCC_OS167mW39dQI5SWwWxqHl4/edit#gid=0

References:

Raw data for each character:

https://docs.google.com/spreadsheets/d/16fmsoqDoQaR1eteVk2uuzIH2DB4iQHVrqiG8VRbRA7Q/edit#gid=123650910

Other attributes for each character:

http://kuroganehammer.com/Ultimate/Attributes

Recovery videos:

https://www.youtube.com/watch?v=Tg8oXDob960

https://www.youtube.com/watch?v=7gXXLdgorDw&t=108s

Tier list:

https://i.imgur.com/IECViTD.jpg

MATLAB

Source code from MATLAB:

Any variable with the o subscript is for the offense category, the d subscript is for the defense category, and the m subscript is for the movement category. b is our raw data (the rows of b are the attributes and the columns of b are the characters), a is transpose(b), A is b*a, d is a vector based on the values given for each character in the tier list, B is b*d, and x is our least squares solution given by A\B.

```
bo =

  Columns 1 through 9

    6.4932   10.3448    7.2289   14.3478   11.6667    5.1316    4.0500    4.9412    7.2727
   16.8000   15.8824   20.0000   18.1818   16.5517   20.8696   18.2609   20.6897   16.8750
   11.3793   15.7895   18.8571   20.0000   13.5484   15.0000   17.7778   11.5385   13.3333
   15.5556   15.0000   18.0000   16.3636   13.0435   18.0000   17.7778   20.0000   23.0769
   12.9730   21.1765   15.0000   14.6341   16.5000   12.2034   11.6129   18.8571    5.3333
   22.5957   24.4444   14.7273   17.5000   17.8846   19.1489   18.6667   20.3774   21.9512
   21.5385   23.2653    7.7922    6.4286   18.2609   20.0000   17.4545   19.0909   21.5385
   16.7442   18.5455   17.1429   16.3636   14.6939   15.8491   16.1538    2.7692   24.3243
   10.6667   17.3684   17.3684   13.3333   13.6364   11.1111   14.2105    5.5263   16.0000
   14.2373   17.4545   11.7647    5.0847   20.9302    6.3830    6.6977    7.0244   20.8696
   19.0909   25.1613   14.0000   20.4878    6.0000   19.5000   16.2500    5.5814   19.0909
   14.0000   21.0811   15.2542    6.1538   20.0000   14.5946   17.1429   13.8462   25.3846
    8.9189   17.7778   13.6709   17.5000    2.8929    2.2222    3.6735   16.5957   21.4286
```

Columns 10 through 18

6.7164	5.7521	10.2857	5.3571	13.2000	10.2857	5.5946	4.9565	7.6438
17.6471	18.6207	25.0000	13.3333	21.0811	29.4375	7.5000	20.0000	19.6800
15.0000	16.9231	22.5000	16.2162	16.5000	9.3750	7.5000	14.8966	15.3103
24.5455	17.6471	20.0000	18.2609	10.6667	21.0000	10.3846	15.7143	18.2222
6.1538	16.2162	18.4615	9.7297	13.0909	15.3659	12.3529	20.5714	18.6486
24.4444	20.3390	19.5918	24.0000	20.0000	27.3913	10.9091	15.9184	26.6809
15.0000	18.6667	20.0000	23.1818	23.1579	23.5102	17.3077	4.7619	25.2308
13.3333	22.5000	13.7500	3.3333	13.3333	34.0500	7.0588	19.4595	19.6744
18.8571	9.2308	15.3488	16.2500	7.6596	16.2667	7.3469	6.0000	12.5333
8.4615	29.3333	15.0000	15.7895	19.0244	22.5000	7.9412	24.4898	17.8983
25.7143	22.2857	21.0811	13.5849	25.9091	30.0000	12.1622	24.4898	25.6364
9.0909	19.3548	13.8462	8.0000	20.4545	26.9143	5.5814	18.8889	20.4000
15.5556	19.0909	2.3077	7.8947	12.4675	13.3333	11.1111	21.8182	19.2273

Columns 19 through 27

4.5000	4.1087	12.4528	9.0566	4.9831	25.3846	5.7273	20.4545	17.7273
20.0000	15.0000	21.8182	20.0000	21.8182	21.5385	18.8571	22.7273	19.8182
13.0435	8.2759	16.3636	14.5455	15.4839	15.1579	14.4828	18.9474	16.4211
20.0000	25.1613	26.0870	22.1739	24.0000	24.0000	13.0435	31.4286	25.7143
14.1176	13.8462	14.6939	13.3469	19.4118	24.3243	18.9474	17.3333	16.0000
9.7959	20.0000	21.1765	17.6471	3.7500	20.8696	23.0769	22.6415	20.3774
21.0000	15.9184	17.5862	14.6897	10.4348	23.6066	8.8235	10.3448	10.3448
9.6000	19.5652	18.5455	15.2727	9.7959	15.2542	22.3256	16.4516	13.7419
10.7692	5.0000	11.6327	10.4082	22.2857	17.5610	4.8980	11.3333	8.8000
5.3846	4.8980	18.6486	17.0270	15.0000	24.5455	20.0000	22.7586	18.6207
2.7778	21.0811	19.2308	18.1538	16.5517	31.7143	20.0000	20.5714	18.6857
9.2308	16.3636	17.3333	15.2000	7.1186	23.6364	18.4615	13.1707	11.1220
16.5957	15.2941	15.2542	14.4407	9.1139	25.9091	17.8723	17.6471	16.7059

Columns 28 through 36

7.5556	5.2941	4.5324	4.3902	8.7097	7.7647	6.6667	6.3380	4.0206
19.4595	5.8537	15.3846	17.1429	20.0000	12.8571	18.2927	16.6667	2.6667
12.7273	12.3529	8.8235	9.3333	13.3333	23.5135	18.4615	15.7895	13.1250
12.8571	16.6667	14.4000	16.5517	14.1176	24.0000	17.1429	18.6207	10.6452

16.2162	13.1250	17.3684	9.4118	13.7500	16.0976	17.8723	13.7143	16.7442
25.7143	23.4146	11.1111	10.3125	18.4615	18.0822	15.9036	18.3673	16.2712
25.2632	6.1224	9.4118	5.0000	17.8947	15.5556	14.5714	12.0000	17.2881
24.3243	21.0811	18.0000	11.7073	12.7869	19.0909	16.2857	19.5000	16.0000
5.7143	13.9535	5.0000	9.4118	8.0000	12.2034	7.6271	14.6341	2.9268
15.3191	4.0000	7.8261	9.1304	11.3514	13.0435	11.6949	12.0000	15.0000
4.6154	4.7059	18.0000	18.0000	14.6939	22.8571	15.5556	10.2857	8.3721
4.2857	9.2308	6.1224	11.4706	16.9565	17.8723	11.7857	14.4828	17.6471
13.4694	13.3333	16.6667	8.6842	5.1064	10.1695	16.6667	5.4545	9.5238

Columns 37 through 45

7.5000	6.1644	7.8947	6.0000	4.3540	13.3333	11.6505	9.0000	6.4865
17.8378	16.6667	26.4000	12.0000	3.7500	18.8571	11.2500	14.5455	18.6207
14.5455	12.4138	14.5455	9.2308	15.7895	6.3158	11.2500	11.5385	12.0000
20.0000	18.3333	20.0000	13.3333	16.2162	12.4138	16.3636	21.4286	19.0909
20.0000	4.5000	21.0811	7.3469	14.5455	11.3514	11.7073	13.5484	16.0000
14.7826	13.2000	20.0000	17.8723	19.4805	35.6923	11.1111	16.6667	17.5000
14.3478	6.7925	12.8571	4.6154	14.3284	34.6667	9.5238	17.5000	18.5714
16.0000	17.3077	17.2881	13.3333	15.0000	27.7895	11.1111	6.6667	16.9565
18.4615	7.5000	5.4545	14.6939	18.4615	2.2642	10.0000	12.3913	10.9756
17.3333	13.0435	18.2927	4.0000	17.5610	19.4286	13.3333	12.0000	20.5263
21.3333	18.0000	18.4615	22.7027	25.9459	21.5000	12.2449	17.3077	13.7143
17.3333	11.6667	20.0000	12.3077	6.8182	25.4118	12.6316	4.7059	14.2373
16.4706	16.2500	5.3571	10.6667	19.1489	16.9412	12.0000	10.4348	24.0000

Columns 46 through 54

6.3158	3.5294	6.8571	4.5161	8.5574	6.3303	4.4531	4.0769	8.3544
10.5882	16.3636	6.8571	20.6250	23.0270	12.9730	15.5625	18.8889	21.1765
17.1429	7.6596	19.6154	20.6897	33.0000	13.4483	8.4375	12.4138	13.1250
13.3333	19.4595	10.4348	21.3158	20.0625	19.2000	10.9091	17.7778	25.2632
17.8378	13.6364	4.0000	15.7895	12.7500	18.1818	17.1429	18.8571	14.6341
21.9512	20.8163	12.7778	16.6071	26.4706	33.4884	17.1429	16.1194	16.2712
15.3191	4.5283	5.2941	18.9474	26.8085	26.8085	16.4706	17.8723	7.1186
22.3256	7.3469	13.0769	14.1176	24.1463	18.5714	15.9184	18.1395	16.9811
17.1429	15.4286	6.8571	15.8824	20.2326	8.0000	12.6923	13.3333	10.0000
13.5000	10.7692	12.7500	18.9474	8.6441	8.3333	15.5556	8.1818	6.5217
20.4545	15.4286	6.8182	16.8750	18.8889	9.7297	9.0000	18.4615	22.7027

18.9474	18.5714	3.3962	15.3846	17.1429	7.3171	4.3902	15.4286	21.8182
16.9811	17.3333	13.1250	20.0000	15.9184	11.1111	9.4118	15.9184	5.1724

Columns 55 through 63

5.5814	6.3582	6.5753	4.4503	5.0000	7.6271	5.0667	9.9083	7.7064
23.6364	7.1186	16.0000	14.0625	18.8372	15.4839	15.4839	17.7391	17.7391
16.2162	2.7273	16.2500	11.2500	15.3846	11.2500	15.5556	19.4595	19.4595
21.0000	13.0769	13.8462	17.1429	18.3871	9.0000	17.7778	15.5556	15.5556
14.6667	17.8378	5.3333	14.6341	18.7500	5.1064	13.9535	18.0000	18.0000
5.8442	15.2381	18.4615	16.8421	11.6418	12.0000	8.1818	23.3333	21.3333
7.6364	15.2381	17.1429	17.3077	12.0896	6.7925	10.5263	23.1818	23.1818
16.1538	13.2353	14.4444	16.0714	10.2439	18.3051	6.3158	23.4146	23.4146
14.6341	5.8824	11.7647	17.6923	8.6441	11.1628	12.0000	13.7143	14.4444
11.1628	13.6364	18.2400	22.7273	11.7073	15.3488	13.6364	23.3333	20.0000
21.0811	14.6939	17.7000	23.0769	13.8889	18.6667	18.2927	23.4146	23.4146
4.0678	5.8824	16.6667	17.3333	11.6667	24.0000	9.2308	10.2857	11.1429
16.6667	11.3793	8.5714	11.1864	11.5000	2.6786	12.2449	16.0000	16.0000

Columns 64 through 72

4.8980	5.2555	3.7278	5.1570	5.0526	3.7297	8.5714	6.3158	6.5934
18.8571	18.5294	6.9643	15.8824	24.3750	24.0000	18.5714	15.0000	21.6667
15.0000	15.4286	11.6129	12.4138	18.6207	16.2162	14.6809	14.1176	15.8824
10.5000	15.0000	12.8571	11.6129	18.0000	7.1287	18.5714	24.3750	18.6207
14.6667	3.6735	14.2857	13.7143	18.4615	3.8889	15.0000	14.6341	19.0244
11.8182	16.2857	14.5455	18.8235	22.6415	17.4194	19.5738	22.6667	23.5294
16.2500	17.6471	15.9375	15.7895	21.7021	17.4545	12.6761	15.3191	21.7021
13.4694	17.5000	14.1176	15.3061	16.8421	17.4545	17.1429	13.9535	16.8421
12.6316	9.1304	15.0000	13.5484	18.4615	5.7143	16.0000	18.1818	19.0244
15.8491	12.8571	1.9672	14.6341	10.2439	18.4615	14.7368	10.7692	17.7273
18.5714	16.0976	5.8235	16.6667	20.9302	18.4615	23.2653	15.4286	19.0244
15.0000	14.5946	14.4828	9.0698	21.5385	18.4615	11.8310	14.2857	15.4839
12.1875	3.5294	15.0000	11.8033	15.2727	15.0000	11.8033	13.3333	20.4545

Column 73

5.2364
12.2222

```
   11.6129
   19.2857
    8.9362
   11.7647
   22.3529
   16.0000
   17.8723
    3.8298
   10.8197
   22.8947
   10.9091

>> ao=transpose(bo)

ao =

  Columns 1 through 9

    6.4932   16.8000   11.3793   15.5556   12.9730   22.5957   21.5385   16.7442   10.6667
   10.3448   15.8824   15.7895   15.0000   21.1765   24.4444   23.2653   18.5455   17.3684
    7.2289   20.0000   18.8571   18.0000   15.0000   14.7273    7.7922   17.1429   17.3684
   14.3478   18.1818   20.0000   16.3636   14.6341   17.5000    6.4286   16.3636   13.3333
   11.6667   16.5517   13.5484   13.0435   16.5000   17.8846   18.2609   14.6939   13.6364
    5.1316   20.8696   15.0000   18.0000   12.2034   19.1489   20.0000   15.8491   11.1111
    4.0500   18.2609   17.7778   17.7778   11.6129   18.6667   17.4545   16.1538   14.2105
    4.9412   20.6897   11.5385   20.0000   18.8571   20.3774   19.0909    2.7692    5.5263
    7.2727   16.8750   13.3333   23.0769    5.3333   21.9512   21.5385   24.3243   16.0000
    6.7164   17.6471   15.0000   24.5455    6.1538   24.4444   15.0000   13.3333   18.8571
    5.7521   18.6207   16.9231   17.6471   16.2162   20.3390   18.6667   22.5000    9.2308
   10.2857   25.0000   22.5000   20.0000   18.4615   19.5918   20.0000   13.7500   15.3488
    5.3571   13.3333   16.2162   18.2609    9.7297   24.0000   23.1818    3.3333   16.2500
   13.2000   21.0811   16.5000   10.6667   13.0909   20.0000   23.1579   13.3333    7.6596
   10.2857   29.4375    9.3750   21.0000   15.3659   27.3913   23.5102   34.0500   16.2667
    5.5946    7.5000    7.5000   10.3846   12.3529   10.9091   17.3077    7.0588    7.3469
    4.9565   20.0000   14.8966   15.7143   20.5714   15.9184    4.7619   19.4595    6.0000
    7.6438   19.6800   15.3103   18.2222   18.6486   26.6809   25.2308   19.6744   12.5333
    4.5000   20.0000   13.0435   20.0000   14.1176    9.7959   21.0000    9.6000   10.7692
    4.1087   15.0000    8.2759   25.1613   13.8462   20.0000   15.9184   19.5652    5.0000
   12.4528   21.8182   16.3636   26.0870   14.6939   21.1765   17.5862   18.5455   11.6327
```

9.0566	20.0000	14.5455	22.1739	13.3469	17.6471	14.6897	15.2727	10.4082
4.9831	21.8182	15.4839	24.0000	19.4118	3.7500	10.4348	9.7959	22.2857
25.3846	21.5385	15.1579	24.0000	24.3243	20.8696	23.6066	15.2542	17.5610
5.7273	18.8571	14.4828	13.0435	18.9474	23.0769	8.8235	22.3256	4.8980
20.4545	22.7273	18.9474	31.4286	17.3333	22.6415	10.3448	16.4516	11.3333
17.7273	19.8182	16.4211	25.7143	16.0000	20.3774	10.3448	13.7419	8.8000
7.5556	19.4595	12.7273	12.8571	16.2162	25.7143	25.2632	24.3243	5.7143
5.2941	5.8537	12.3529	16.6667	13.1250	23.4146	6.1224	21.0811	13.9535
4.5324	15.3846	8.8235	14.4000	17.3684	11.1111	9.4118	18.0000	5.0000
4.3902	17.1429	9.3333	16.5517	9.4118	10.3125	5.0000	11.7073	9.4118
8.7097	20.0000	13.3333	14.1176	13.7500	18.4615	17.8947	12.7869	8.0000
7.7647	12.8571	23.5135	24.0000	16.0976	18.0822	15.5556	19.0909	12.2034
6.6667	18.2927	18.4615	17.1429	17.8723	15.9036	14.5714	16.2857	7.6271
6.3380	16.6667	15.7895	18.6207	13.7143	18.3673	12.0000	19.5000	14.6341
4.0206	2.6667	13.1250	10.6452	16.7442	16.2712	17.2881	16.0000	2.9268
7.5000	17.8378	14.5455	20.0000	20.0000	14.7826	14.3478	16.0000	18.4615
6.1644	16.6667	12.4138	18.3333	4.5000	13.2000	6.7925	17.3077	7.5000
7.8947	26.4000	14.5455	20.0000	21.0811	20.0000	12.8571	17.2881	5.4545
6.0000	12.0000	9.2308	13.3333	7.3469	17.8723	4.6154	13.3333	14.6939
4.3540	3.7500	15.7895	16.2162	14.5455	19.4805	14.3284	15.0000	18.4615
13.3333	18.8571	6.3158	12.4138	11.3514	35.6923	34.6667	27.7895	2.2642
11.6505	11.2500	11.2500	16.3636	11.7073	11.1111	9.5238	11.1111	10.0000
9.0000	14.5455	11.5385	21.4286	13.5484	16.6667	17.5000	6.6667	12.3913
6.4865	18.6207	12.0000	19.0909	16.0000	17.5000	18.5714	16.9565	10.9756
6.3158	10.5882	17.1429	13.3333	17.8378	21.9512	15.3191	22.3256	17.1429
3.5294	16.3636	7.6596	19.4595	13.6364	20.8163	4.5283	7.3469	15.4286
6.8571	6.8571	19.6154	10.4348	4.0000	12.7778	5.2941	13.0769	6.8571
4.5161	20.6250	20.6897	21.3158	15.7895	16.6071	18.9474	14.1176	15.8824
8.5574	23.0270	33.0000	20.0625	12.7500	26.4706	26.8085	24.1463	20.2326
6.3303	12.9730	13.4483	19.2000	18.1818	33.4884	26.8085	18.5714	8.0000
4.4531	15.5625	8.4375	10.9091	17.1429	17.1429	16.4706	15.9184	12.6923
4.0769	18.8889	12.4138	17.7778	18.8571	16.1194	17.8723	18.1395	13.3333
8.3544	21.1765	13.1250	25.2632	14.6341	16.2712	7.1186	16.9811	10.0000
5.5814	23.6364	16.2162	21.0000	14.6667	5.8442	7.6364	16.1538	14.6341
6.3582	7.1186	2.7273	13.0769	17.8378	15.2381	15.2381	13.2353	5.8824
6.5753	16.0000	16.2500	13.8462	5.3333	18.4615	17.1429	14.4444	11.7647
4.4503	14.0625	11.2500	17.1429	14.6341	16.8421	17.3077	16.0714	17.6923
5.0000	18.8372	15.3846	18.3871	18.7500	11.6418	12.0896	10.2439	8.6441
7.6271	15.4839	11.2500	9.0000	5.1064	12.0000	6.7925	18.3051	11.1628

5.0667	15.4839	15.5556	17.7778	13.9535	8.1818	10.5263	6.3158	12.0000
9.9083	17.7391	19.4595	15.5556	18.0000	23.3333	23.1818	23.4146	13.7143
7.7064	17.7391	19.4595	15.5556	18.0000	21.3333	23.1818	23.4146	14.4444
4.8980	18.8571	15.0000	10.5000	14.6667	11.8182	16.2500	13.4694	12.6316
5.2555	18.5294	15.4286	15.0000	3.6735	16.2857	17.6471	17.5000	9.1304
3.7278	6.9643	11.6129	12.8571	14.2857	14.5455	15.9375	14.1176	15.0000
5.1570	15.8824	12.4138	11.6129	13.7143	18.8235	15.7895	15.3061	13.5484
5.0526	24.3750	18.6207	18.0000	18.4615	22.6415	21.7021	16.8421	18.4615
3.7297	24.0000	16.2162	7.1287	3.8889	17.4194	17.4545	17.4545	5.7143
8.5714	18.5714	14.6809	18.5714	15.0000	19.5738	12.6761	17.1429	16.0000
6.3158	15.0000	14.1176	24.3750	14.6341	22.6667	15.3191	13.9535	18.1818
6.5934	21.6667	15.8824	18.6207	19.0244	23.5294	21.7021	16.8421	19.0244
5.2364	12.2222	11.6129	19.2857	8.9362	11.7647	22.3529	16.0000	17.8723

Columns 10 through 13

14.2373	19.0909	14.0000	8.9189
17.4545	25.1613	21.0811	17.7778
11.7647	14.0000	15.2542	13.6709
5.0847	20.4878	6.1538	17.5000
20.9302	6.0000	20.0000	2.8929
6.3830	19.5000	14.5946	2.2222
6.6977	16.2500	17.1429	3.6735
7.0244	5.5814	13.8462	16.5957
20.8696	19.0909	25.3846	21.4286
8.4615	25.7143	9.0909	15.5556
29.3333	22.2857	19.3548	19.0909
15.0000	21.0811	13.8462	2.3077
15.7895	13.5849	8.0000	7.8947
19.0244	25.9091	20.4545	12.4675
22.5000	30.0000	26.9143	13.3333
7.9412	12.1622	5.5814	11.1111
24.4898	24.4898	18.8889	21.8182
17.8983	25.6364	20.4000	19.2273
5.3846	2.7778	9.2308	16.5957
4.8980	21.0811	16.3636	15.2941
18.6486	19.2308	17.3333	15.2542
17.0270	18.1538	15.2000	14.4407
15.0000	16.5517	7.1186	9.1139

24.5455	31.7143	23.6364	25.9091
20.0000	20.0000	18.4615	17.8723
22.7586	20.5714	13.1707	17.6471
18.6207	18.6857	11.1220	16.7059
15.3191	4.6154	4.2857	13.4694
4.0000	4.7059	9.2308	13.3333
7.8261	18.0000	6.1224	16.6667
9.1304	18.0000	11.4706	8.6842
11.3514	14.6939	16.9565	5.1064
13.0435	22.8571	17.8723	10.1695
11.6949	15.5556	11.7857	16.6667
12.0000	10.2857	14.4828	5.4545
15.0000	8.3721	17.6471	9.5238
17.3333	21.3333	17.3333	16.4706
13.0435	18.0000	11.6667	16.2500
18.2927	18.4615	20.0000	5.3571
4.0000	22.7027	12.3077	10.6667
17.5610	25.9459	6.8182	19.1489
19.4286	21.5000	25.4118	16.9412
13.3333	12.2449	12.6316	12.0000
12.0000	17.3077	4.7059	10.4348
20.5263	13.7143	14.2373	24.0000
13.5000	20.4545	18.9474	16.9811
10.7692	15.4286	18.5714	17.3333
12.7500	6.8182	3.3962	13.1250
18.9474	16.8750	15.3846	20.0000
8.6441	18.8889	17.1429	15.9184
8.3333	9.7297	7.3171	11.1111
15.5556	9.0000	4.3902	9.4118
8.1818	18.4615	15.4286	15.9184
6.5217	22.7027	21.8182	5.1724
11.1628	21.0811	4.0678	16.6667
13.6364	14.6939	5.8824	11.3793
18.2400	17.7000	16.6667	8.5714
22.7273	23.0769	17.3333	11.1864
11.7073	13.8889	11.6667	11.5000
15.3488	18.6667	24.0000	2.6786
13.6364	18.2927	9.2308	12.2449
23.3333	23.4146	10.2857	16.0000

```
   20.0000   23.4146   11.1429   16.0000
   15.8491   18.5714   15.0000   12.1875
   12.8571   16.0976   14.5946    3.5294
    1.9672    5.8235   14.4828   15.0000
   14.6341   16.6667    9.0698   11.8033
   10.2439   20.9302   21.5385   15.2727
   18.4615   18.4615   18.4615   15.0000
   14.7368   23.2653   11.8310   11.8033
   10.7692   15.4286   14.2857   13.3333
   17.7273   19.0244   15.4839   20.4545
    3.8298   10.8197   22.8947   10.9091

>> Ao=bo*ao

Ao =

   1.0e+04 *

  Columns 1 through 9

    0.5045    0.9678    0.8050    0.9911    0.8049    1.0375    0.8869    0.8936    0.6583
    0.9678    2.3446    1.8673    2.2666    1.8414    2.3389    2.0460    2.0722    1.5377
    0.8050    1.8673    1.6697    1.8918    1.5357    1.9660    1.7097    1.7342    1.3351
    0.9911    2.2666    1.8918    2.4092    1.8808    2.3792    2.0377    2.0684    1.6069
    0.8049    1.8414    1.5357    1.8808    1.6610    1.9585    1.7107    1.7120    1.2885
    1.0375    2.3389    1.9660    2.3792    1.9585    2.6978    2.2938    2.2783    1.6320
    0.8869    2.0460    1.7097    2.0377    1.7107    2.2938    2.1507    1.9529    1.4291
    0.8936    2.0722    1.7342    2.0684    1.7120    2.2783    1.9529    2.1039    1.4313
    0.6583    1.5377    1.3351    1.6069    1.2885    1.6320    1.4291    1.4313    1.2297
    0.8208    1.8273    1.5140    1.8047    1.5241    1.9460    1.7010    1.7318    1.2432
    1.0052    2.2860    1.8929    2.3001    1.8723    2.4125    2.0649    2.1486    1.6055
    0.8065    1.8709    1.5253    1.8546    1.5133    2.0007    1.7443    1.7848    1.2855
    0.7436    1.6737    1.4140    1.7355    1.4428    1.8200    1.5651    1.6053    1.1896

  Columns 10 through 13

    0.8208    1.0052    0.8065    0.7436
    1.8273    2.2860    1.8709    1.6737
    1.5140    1.8929    1.5253    1.4140
```

1.8047	2.3001	1.8546	1.7355
1.5241	1.8723	1.5133	1.4428
1.9460	2.4125	2.0007	1.8200
1.7010	2.0649	1.7443	1.5651
1.7318	2.1486	1.7848	1.6053
1.2432	1.6055	1.2855	1.1896
1.6892	1.9108	1.5500	1.4274
1.9108	2.5120	1.9333	1.7581
1.5500	1.9333	1.7382	1.3990
1.4274	1.7581	1.3990	1.4794

d =

3.1000
2.7000
3.5000
2.1000
3.3000
0.6000
4.6000
4.3000
1.7000
3.1000
2.2000
1.1000
5.0000
2.4000
1.4000
1.2000
1.5000
1.6000
4.9000
2.3000
3.3000
4.6000
3.8000
1.8000
2.4000
3.9000

3.8000
1.6000
2.6000
2.1000
3.6000
4.0000
3.8000
3.9000
2.5000
3.4000
0.6000
1.8000
2.0000
2.3000
2.3000
4.8000
2.8000
3.6000
2.0000
4.8000
2.0000
3.5000
1.9000
1.6000
0.1000
4.3000
1.6000
2.4000
2.0000
4.2000
2.8000
1.2000
4.0000
0.8000
1.4000
1.2000
1.6000
3.4000
1.9000

```
    1.5000
    4.5000
    1.7000
    2.6000
    0.7000
    1.1000
    1.9000
    0.7000

>> Bo=bo*d

Bo =

   1.0e+03 *

    1.4012
    3.1792
    2.6899
    3.2790
    2.6830
    3.4139
    2.9550
    2.9029
    2.1888
    2.6221
    3.1501
    2.5999
    2.4465

>> xo=Ao\Bo

xo =

    0.0041
    0.0137
    0.0495
    0.0647
    0.0310
    0.0459
```

```
    0.0019
   -0.0385
   -0.0253
    0.0371
   -0.0419
    0.0034
    0.008700

bd =

  Columns 1 through 15

   -20   -21   -21   -21   -21   -21   -18   -19   -20   -20   -21   -21   -21   -23   -20
   -29   -30   -30   -34   -30   -30   -26   -28   -29   -29   -30   -30   -30   -32   -29
   -34   -35   -35   -39   -35   -35   -32   -33   -34   -34   -35   -35   -35   -37   -34
   -52   -48   -49   -56   -58   -62   -38   -50   -57   -59   -42   -74   -61   -46   -58
   -71   -66   -69   -80   -79   -82   -54   -69   -81   -77   -62  -102   -86   -63   -81
   -77   -73   -74   -86   -89   -91   -59   -76   -88   -86   -66  -112   -92   -67   -86
   -87   -82   -82   -96  -107  -102   -64   -85   -96   -99   -72  -122  -103   -75   -96
  -102   -88   -95  -113  -116  -134   -70   -97  -113  -119   -85  -152  -124   -88  -115
  -116  -109  -108  -130  -130  -141   -75  -109  -127  -134   -94  -174  -141   -95  -129
    98   127   104   108   104    79    77    79    97    94   104    68    89   135    92

  Columns 16 through 30

   -18   -21   -20   -19   -20   -20   -20   -19   -23   -20   -20   -20   -21   -19   -20
   -26   -30   -29   -28   -29   -29   -29   -28   -32   -29   -29   -29   -30   -28   -29
   -32   -35   -34   -33   -34   -34   -34   -33   -37   -34   -34   -34   -35   -33   -34
   -44   -56   -52   -42   -43   -52   -52   -45   -46   -51   -44   -44   -59   -47   -52
   -63   -77   -71   -60   -61   -69   -69   -63   -66   -70   -62   -62   -84   -66   -73
   -67   -86   -81   -65   -67   -79   -79   -70   -73   -79   -68   -68   -93   -74   -82
   -74   -96   -87   -70   -73   -85   -85   -76   -79   -85   -74   -74  -100   -80   -89
   -85  -114  -100   -80   -85   -99   -99   -90   -91  -101   -87   -87  -123   -94  -110
   -93  -129  -116   -88   -94  -116  -116  -100  -103  -114   -97   -97  -132  -101  -119
    78    85    98    62    82    90    90    88   118    77    95    95    75    80    96

  Columns 31 through 45

   -18   -21   -21   -21   -19   -21   -23   -19   -20   -19   -23   -20   -20   -20   -19
```

```
   -26   -30   -30   -30   -28   -30   -32   -28   -29   -28   -32   -29   -29   -29   -28
   -32   -35   -35   -35   -33   -35   -37   -33   -34   -33   -37   -34   -34   -34   -33
   -45   -50   -47   -49   -53   -56   -49   -44   -56   -50   -43   -57   -48   -50   -56
   -65   -66   -64   -66   -79   -76   -70   -62   -75   -66   -58   -78   -65   -68   -76
   -73   -75   -75   -76   -87   -84   -78   -68   -85   -74   -66   -87   -74   -75   -85
   -80   -82   -81   -82   -90   -93   -85   -74   -95   -81   -72   -96   -91   -83   -93
   -90   -97   -99   -96  -105  -110   -99   -86  -117   -99   -86  -118   -98  -100  -111
   -98  -104  -108  -108  -114  -124  -105   -96  -126  -107   -96  -132  -108  -111  -123
    80   107   106   107    75    96   116    90    94    86   127    79    92   106    91

  Columns 46 through 60

   -20   -20   -20   -20   -21   -18   -19   -20   -20   -20   -20   -20   -21   -20   -20
   -29   -29   -29   -29   -30   -26   -28   -29   -29   -29   -29   -29   -30   -29   -29
   -34   -34   -34   -34   -35   -32   -33   -34   -34   -34   -34   -34   -35   -34   -34
   -44   -57   -45   -57   -62   -49   -41   -40   -50   -52   -49   -55   -51   -49   -48
   -61   -80   -63   -80   -86   -62   -60   -58   -69   -73   -70   -78   -73   -68   -67
   -68   -87   -70   -88   -94   -64   -67   -65   -77   -82   -77   -87   -79   -77   -77
   -73   -96   -76   -96  -105   -72   -71   -70   -84   -87   -82   -96   -86   -83   -82
   -84  -111   -84  -107  -124   -81   -78   -80   -99  -103   -93  -111  -103   -95   -93
   -93  -128   -98  -124  -144   -89   -85   -86  -107  -114  -105  -130  -114  -107  -108
    92    92   102    96    82    87    88    94   100   104    91    95    95    97   108

  Columns 61 through 73

   -20   -21   -21   -20   -20   -20   -20   -23   -21   -23   -20   -21   -20
   -29   -30   -30   -29   -29   -31   -29   -32   -30   -32   -29   -30   -29
   -34   -35   -35   -34   -34   -36   -34   -37   -35   -37   -34   -35   -34
   -49   -49   -49   -47   -49   -45   -50   -46   -46   -46   -58   -44   -42
   -66   -67   -67   -64   -67   -63   -70   -64   -62   -64   -80   -63   -58
   -78   -73   -73   -72   -75   -69   -76   -73   -69   -74   -91   -72   -66
   -83   -80   -80   -79   -81   -74   -84   -78   -76   -78   -97   -76   -69
  -100   -89   -89   -94   -97   -85  -100   -90   -90   -88  -115   -86   -81
  -113  -102  -102  -104  -107   -93  -112  -103  -104  -102  -133   -96   -88
    86   103   103   100    98    81    94   107   107   133    88   116   112

>> ad=transpose(bd)

ad =
```

-20	-29	-34	-52	-71	-77	-87	-102	-116	98
-21	-30	-35	-48	-66	-73	-82	-88	-109	127
-21	-30	-35	-49	-69	-74	-82	-95	-108	104
-21	-34	-39	-56	-80	-86	-96	-113	-130	108
-21	-30	-35	-58	-79	-89	-107	-116	-130	104
-21	-30	-35	-62	-82	-91	-102	-134	-141	79
-18	-26	-32	-38	-54	-59	-64	-70	-75	77
-19	-28	-33	-50	-69	-76	-85	-97	-109	79
-20	-29	-34	-57	-81	-88	-96	-113	-127	97
-20	-29	-34	-59	-77	-86	-99	-119	-134	94
-21	-30	-35	-42	-62	-66	-72	-85	-94	104
-21	-30	-35	-74	-102	-112	-122	-152	-174	68
-21	-30	-35	-61	-86	-92	-103	-124	-141	89
-23	-32	-37	-46	-63	-67	-75	-88	-95	135
-20	-29	-34	-58	-81	-86	-96	-115	-129	92
-18	-26	-32	-44	-63	-67	-74	-85	-93	78
-21	-30	-35	-56	-77	-86	-96	-114	-129	85
-20	-29	-34	-52	-71	-81	-87	-100	-116	98
-19	-28	-33	-42	-60	-65	-70	-80	-88	62
-20	-29	-34	-43	-61	-67	-73	-85	-94	82
-20	-29	-34	-52	-69	-79	-85	-99	-116	90
-20	-29	-34	-52	-69	-79	-85	-99	-116	90
-19	-28	-33	-45	-63	-70	-76	-90	-100	88
-23	-32	-37	-46	-66	-73	-79	-91	-103	118
-20	-29	-34	-51	-70	-79	-85	-101	-114	77
-20	-29	-34	-44	-62	-68	-74	-87	-97	95
-20	-29	-34	-44	-62	-68	-74	-87	-97	95
-21	-30	-35	-59	-84	-93	-100	-123	-132	75
-19	-28	-33	-47	-66	-74	-80	-94	-101	80
-20	-29	-34	-52	-73	-82	-89	-110	-119	96
-18	-26	-32	-45	-65	-73	-80	-90	-98	80
-21	-30	-35	-50	-66	-75	-82	-97	-104	107
-21	-30	-35	-47	-64	-75	-81	-99	-108	106
-21	-30	-35	-49	-66	-76	-82	-96	-108	107
-19	-28	-33	-53	-79	-87	-90	-105	-114	75
-21	-30	-35	-56	-76	-84	-93	-110	-124	96
-23	-32	-37	-49	-70	-78	-85	-99	-105	116
-19	-28	-33	-44	-62	-68	-74	-86	-96	90

-20	-29	-34	-56	-75	-85	-95	-117	-126	94
-19	-28	-33	-50	-66	-74	-81	-99	-107	86
-23	-32	-37	-43	-58	-66	-72	-86	-96	127
-20	-29	-34	-57	-78	-87	-96	-118	-132	79
-20	-29	-34	-48	-65	-74	-91	-98	-108	92
-20	-29	-34	-50	-68	-75	-83	-100	-111	106
-19	-28	-33	-56	-76	-85	-93	-111	-123	91
-20	-29	-34	-44	-61	-68	-73	-84	-93	92
-20	-29	-34	-57	-80	-87	-96	-111	-128	92
-20	-29	-34	-45	-63	-70	-76	-84	-98	102
-20	-29	-34	-57	-80	-88	-96	-107	-124	96
-21	-30	-35	-62	-86	-94	-105	-124	-144	82
-18	-26	-32	-49	-62	-64	-72	-81	-89	87
-19	-28	-33	-41	-60	-67	-71	-78	-85	88
-20	-29	-34	-40	-58	-65	-70	-80	-86	94
-20	-29	-34	-50	-69	-77	-84	-99	-107	100
-20	-29	-34	-52	-73	-82	-87	-103	-114	104
-20	-29	-34	-49	-70	-77	-82	-93	-105	91
-20	-29	-34	-55	-78	-87	-96	-111	-130	95
-21	-30	-35	-51	-73	-79	-86	-103	-114	95
-20	-29	-34	-49	-68	-77	-83	-95	-107	97
-20	-29	-34	-48	-67	-77	-82	-93	-108	108
-20	-29	-34	-49	-66	-78	-83	-100	-113	86
-21	-30	-35	-49	-67	-73	-80	-89	-102	103
-21	-30	-35	-49	-67	-73	-80	-89	-102	103
-20	-29	-34	-47	-64	-72	-79	-94	-104	100
-20	-29	-34	-49	-67	-75	-81	-97	-107	98
-20	-31	-36	-45	-63	-69	-74	-85	-93	81
-20	-29	-34	-50	-70	-76	-84	-100	-112	94
-23	-32	-37	-46	-64	-73	-78	-90	-103	107
-21	-30	-35	-46	-62	-69	-76	-90	-104	107
-23	-32	-37	-46	-64	-74	-78	-88	-102	133
-20	-29	-34	-58	-80	-91	-97	-115	-133	88
-21	-30	-35	-44	-63	-72	-76	-86	-96	116
-20	-29	-34	-42	-58	-66	-69	-81	-88	112

>> Ad=bd*ad

Ad =

Columns 1 through 7

30139	43520	50997	74336	103043	114433	125244
43520	62874	73678	107423	148920	165367	180989
50997	73678	86351	125904	174539	193805	212114
74336	107423	125904	186435	258213	286605	313979
103043	148920	174539	258213	357915	397198	435011
114433	165367	193805	286605	397198	441035	482945
125244	180989	212114	313979	435011	482945	529235
146601	211850	248261	368254	510085	566296	620485
164133	237186	277931	412505	571357	634326	695041
-142130	-205105	-240262	-347704	-481845	-535343	-585926

Columns 8 through 10

146601	164133	-142130
211850	237186	-205105
248261	277931	-240262
368254	412505	-347704
510085	571357	-481845
566296	634326	-535343
620485	695041	-585926
728487	815689	-684695
815689	914102	-767066
-684695	-767066	679405

```
>> Bd=bd*d

Bd =

   1.0e+04 *

   -0.3776
   -0.5463
   -0.6409
   -0.9287
   -1.2880
   -1.4307
```

```
   -1.5680
   -1.8303
   -2.0499
    1.7660

>> xd=Ad\Bd

xd =

    0.1638
    0.4570
   -0.7320
    0.1402
    0.0704
    0.0453
   -0.1159
    0.0552
   -0.0705
   -0.0272

bm =

  Columns 1 through 9

  -34.0000  -38.0000  -34.0000  -59.0000  -48.0000  -34.0000  -36.0000  -36.0000  -47.0000
  -42.0000  -46.0000  -42.0000  -67.0000  -56.0000  -41.0000  -44.0000  -44.0000  -51.0000
  -36.0000  -41.0000  -37.0000  -62.0000  -51.0000  -36.0000  -39.0000  -39.0000  -49.0000
   40.1867   42.7668   35.0263   37.7663   46.7170   39.4332   54.8457   46.5572   37.6750
   45.1353   45.1353   34.5240   41.2121   50.2167   31.3855   41.4736   35.7570   28.7700
   26.9276    7.0862   63.7759   31.1793   29.0534   31.1793   38.9741   75.1138   45.3517
   23.1187   43.6688   30.8250   28.2563   74.4938   41.1000   25.6875   46.2375   43.6688

  Columns 10 through 18

  -37.0000  -35.0000  -32.0000  -36.0000  -40.0000  -39.0000  -36.0000  -39.0000  -34.0000
  -45.0000  -43.0000  -39.0000  -44.0000  -48.0000  -47.0000  -44.0000  -47.0000  -42.0000
  -40.0000  -38.0000  -34.0000  -39.0000  -43.0000  -40.0000  -39.0000  -42.0000  -36.0000
   36.7388   58.2707   29.0212   36.4192   45.0045   34.9350   55.2567   32.6517   31.9163
   37.6252   45.5089   49.7684   38.4472   43.1550   31.0118   43.1550   40.8011   34.5195
```

28.3448	37.5569	30.4707	10.6293	24.8017	37.5569	72.9879	31.1793	12.7552
33.3937	30.8250	102.7500	105.3187	33.3937	30.8250	35.9625	35.9625	12.8438

Columns 19 through 27

-31.0000	-34.0000	-34.0000	-34.0000	-51.0000	-38.0000	-39.0000	-36.0000	-36.0000
-39.0000	-42.0000	-42.0000	-42.0000	-60.0000	-46.0000	-47.0000	-44.0000	-44.0000
-36.0000	-37.0000	-37.0000	-37.0000	-55.0000	-41.0000	-42.0000	-39.0000	-39.0000
43.2007	36.9672	44.8447	44.8447	39.9355	30.5967	51.4892	48.9775	48.9775
38.4472	36.5043	40.0165	40.0165	36.0933	31.0118	49.0585	48.6475	48.6475
66.6103	44.6431	26.9276	26.9276	58.8155	10.6293	46.7690	10.6293	18.4241
53.9438	48.8062	17.9812	17.9812	35.9625	10.2750	53.9438	23.1187	12.8438

Columns 28 through 36

-34.0000	-36.0000	-34.0000	-58.0000	-35.0000	-34.0000	-36.0000	-34.0000	-42.0000
-42.0000	-44.0000	-42.0000	-66.0000	-43.0000	-42.0000	-44.0000	-39.0000	-48.0000
-37.0000	-39.0000	-37.0000	-61.0000	-38.0000	-37.0000	-39.0000	-37.0000	-44.0000
38.3372	47.7217	41.7393	52.7450	37.6750	36.4192	34.4098	40.1867	36.4192
43.9396	38.8582	34.9350	47.0782	47.4892	36.8779	42.3704	37.7373	37.2889
29.7621	46.0603	48.8948	46.0603	51.0207	77.9483	21.2586	46.4147	5.6690
61.6500	61.6500	51.3750	38.5312	61.6500	107.8875	28.2563	17.9812	12.8438

Columns 37 through 45

-39.0000	-36.0000	-46.0000	-37.0000	-39.0000	-56.0000	-39.0000	-37.0000	-52.0000
-45.0000	-44.0000	-54.0000	-47.0000	-47.0000	-68.0000	-46.0000	-45.0000	-60.0000
-42.0000	-39.0000	-49.0000	-39.0000	-42.0000	-60.0000	-42.0000	-40.0000	-55.0000
50.2333	45.8037	37.6750	87.9083	34.1587	36.9215	38.9308	39.3875	43.5203
41.2121	34.5240	43.1550	45.1353	27.4623	32.1701	47.8628	42.3704	39.2318
24.8017	48.8948	53.8552	70.8621	52.4379	56.6897	76.5310	77.2397	59.5241
56.5125	35.9625	53.9438	33.3937	41.1000	87.3375	53.9438	79.6312	48.8062

Columns 46 through 54

-36.0000	-43.0000	-37.0000	-34.0000	-39.0000	-38.0000	-38.0000	-34.0000	-34.0000
-44.0000	-47.0000	-45.0000	-42.0000	-46.0000	-46.0000	-46.0000	-42.0000	-42.0000
-39.0000	-44.0000	-40.0000	-37.0000	-41.0000	-41.0000	-41.0000	-37.0000	-37.0000
35.1633	31.8982	36.5790	42.6070	40.9858	56.2613	52.2427	43.8400	36.0767

```
   47.8628   36.8779   43.1550   38.0735   39.2318   45.1353   46.2935   42.9682   43.7155
   25.5103   71.5707   72.2793   42.5172   36.1397   22.6759   73.6966   24.8017   24.8017
   28.2563   87.3375   28.2563   35.9625   59.0812   17.9812   41.1000   12.8438   38.5312

  Columns 55 through 63

  -34.0000  -39.0000  -58.0000  -39.0000  -36.0000  -40.0000  -34.0000  -34.0000  -34.0000
  -42.0000  -47.0000  -66.0000  -47.0000  -44.0000  -48.0000  -42.0000  -42.0000  -42.0000
  -37.0000  -42.0000  -61.0000  -42.0000  -39.0000  -43.0000  -37.0000  -37.0000  -37.0000
   31.2817   47.4248   38.1773   28.8842   38.1773   35.7570   40.9402   36.5333   40.1867
   34.7482   37.3636   40.8011   39.2318   41.5857   42.3704   43.1550   41.8473   41.8473
   36.8483   33.3052   74.4052   34.0138   49.6034   66.6103   57.3983    7.7948    9.9207
   25.6875   28.2563   71.9250   41.1000   43.6688   35.9625   48.8062   20.5500   20.5500

  Columns 64 through 72

  -36.0000  -36.0000  -37.0000  -34.0000  -34.0000  -37.0000  -40.0000  -43.0000  -38.0000
  -44.0000  -44.0000  -45.0000  -41.0000  -42.0000  -45.0000  -48.0000  -46.0000  -46.0000
  -39.0000  -39.0000  -40.0000  -37.0000  -39.0000  -42.0000  -43.0000  -44.0000  -44.0000
   49.4798   36.4192   40.1867   43.9542   50.2333   34.7067   33.9075   33.7933   26.9433
   43.1550   38.0735   38.0735   45.1353   39.2318   35.1218   35.3086   38.1109   32.8800
   46.7690   36.1397   75.8224   43.9345   43.9345   12.7552   42.5172   57.3983   19.1328
   43.6688   23.1187   38.5312   38.5312   33.3937   12.8438   43.6688   82.2000   23.1187

  Column 73

  -34.0000
  -42.0000
  -21.8552
   39.2733
   37.3636
   50.3121
   48.8062

>> am=transpose(bm)

am =

  -34.0000  -42.0000  -36.0000   40.1867   45.1353   26.9276   23.1187
```

-38.0000	-46.0000	-41.0000	42.7668	45.1353	7.0862	43.6688
-34.0000	-42.0000	-37.0000	35.0263	34.5240	63.7759	30.8250
-59.0000	-67.0000	-62.0000	37.7663	41.2121	31.1793	28.2563
-48.0000	-56.0000	-51.0000	46.7170	50.2167	29.0534	74.4938
-34.0000	-41.0000	-36.0000	39.4332	31.3855	31.1793	41.1000
-36.0000	-44.0000	-39.0000	54.8457	41.4736	38.9741	25.6875
-36.0000	-44.0000	-39.0000	46.5572	35.7570	75.1138	46.2375
-47.0000	-51.0000	-49.0000	37.6750	28.7700	45.3517	43.6688
-37.0000	-45.0000	-40.0000	36.7388	37.6252	28.3448	33.3937
-35.0000	-43.0000	-38.0000	58.2707	45.5089	37.5569	30.8250
-32.0000	-39.0000	-34.0000	29.0212	49.7684	30.4707	102.7500
-36.0000	-44.0000	-39.0000	36.4192	38.4472	10.6293	105.3187
-40.0000	-48.0000	-43.0000	45.0045	43.1550	24.8017	33.3937
-39.0000	-47.0000	-40.0000	34.9350	31.0118	37.5569	30.8250
-36.0000	-44.0000	-39.0000	55.2567	43.1550	72.9879	35.9625
-39.0000	-47.0000	-42.0000	32.6517	40.8011	31.1793	35.9625
-34.0000	-42.0000	-36.0000	31.9163	34.5195	12.7552	12.8438
-31.0000	-39.0000	-36.0000	43.2007	38.4472	66.6103	53.9438
-34.0000	-42.0000	-37.0000	36.9672	36.5043	44.6431	48.8062
-34.0000	-42.0000	-37.0000	44.8447	40.0165	26.9276	17.9812
-34.0000	-42.0000	-37.0000	44.8447	40.0165	26.9276	17.9812
-51.0000	-60.0000	-55.0000	39.9355	36.0933	58.8155	35.9625
-38.0000	-46.0000	-41.0000	30.5967	31.0118	10.6293	10.2750
-39.0000	-47.0000	-42.0000	51.4892	49.0585	46.7690	53.9438
-36.0000	-44.0000	-39.0000	48.9775	48.6475	10.6293	23.1187
-36.0000	-44.0000	-39.0000	48.9775	48.6475	18.4241	12.8438
-34.0000	-42.0000	-37.0000	38.3372	43.9396	29.7621	61.6500
-36.0000	-44.0000	-39.0000	47.7217	38.8582	46.0603	61.6500
-34.0000	-42.0000	-37.0000	41.7393	34.9350	48.8948	51.3750
-58.0000	-66.0000	-61.0000	52.7450	47.0782	46.0603	38.5312
-35.0000	-43.0000	-38.0000	37.6750	47.4892	51.0207	61.6500
-34.0000	-42.0000	-37.0000	36.4192	36.8779	77.9483	107.8875
-36.0000	-44.0000	-39.0000	34.4098	42.3704	21.2586	28.2563
-34.0000	-39.0000	-37.0000	40.1867	37.7373	46.4147	17.9812
-42.0000	-48.0000	-44.0000	36.4192	37.2889	5.6690	12.8438
-39.0000	-45.0000	-42.0000	50.2333	41.2121	24.8017	56.5125
-36.0000	-44.0000	-39.0000	45.8037	34.5240	48.8948	35.9625
-46.0000	-54.0000	-49.0000				

```
>> Am=bm*am

Am =

   1.0e+05 *

    1.1022    1.3196    1.1799   -1.1554   -1.1249   -1.1796   -1.1933
    1.3196    1.5819    1.4131   -1.3907   -1.3528   -1.4147   -1.4284
    1.1799    1.4131    1.2657   -1.2385   -1.2059   -1.2612   -1.2743
   -1.1554   -1.3907   -1.2385    1.3006    1.2246    1.2740    1.2427
   -1.1249   -1.3528   -1.2059    1.2246    1.1989    1.2151    1.2351
   -1.1796   -1.4147   -1.2612    1.2740    1.2151    1.5584    1.4100
   -1.1933   -1.4284   -1.2743    1.2427    1.2351    1.4100    1.6422

>> Bm=bm*d

Bm =

   1.0e+03 *

   -7.2140
   -8.6936
   -7.7736
    7.8234
    7.6151
    7.9565
    8.0325

>> xm=Am\Bm

xm =

    0.3465
   -0.1869
   -0.1480
   -0.0007
    0.0200
    0.0042
    0.0052
```